

The "Transformation" from Marxian "Values" to Competitive "Prices": A Process of Rejection and Replacement

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Abstract. The well-known transformation procedure for transforming from Marxian values to competitive prices is shown to be logically of the form: "Anything" equals "anything else" multiplied by "anything/anything else."

Alternative Systems of Values and Prices. Let $a_0 = [a_{0j}]$ be the row vector of direct labor inputs needed to produce the outputs of n industries; $a = [a_{ij}]$ be the Leontief square matrix whose elements denote the input of the i th good needed to produce the output of the j th industry; $m = [m_i]$ be the column vector of minimum-subsistence goods needed as real wage to cover the cost of production and reproduction of labor. Karl Marx in Volume I of *Capital* assumes that every industry adds to cost outlays on labor and raw materials a constant percentage of the wage payments alone, namely s the "rate of surplus value or labor exploitation."¹ If W is the wage rate, the row vector of Marxian "values," $\pi = [\pi_j]$, is defined by²

$$\begin{aligned}\pi &= Wa_0 + \pi a + sWa_0 = Wa_0[I - a]^{-1}(1 + s) \\ &= WA_0(0)(1 + s) \\ \pi m &= W\end{aligned}\tag{1}$$

An alternative—and incompatible system unless a_{0j}/A_{0j} happen to be identical for all industries—is that provided by the competitive "prices" of bourgeois economics (so-called Walrasian equilibrium) and of Marx's posthumous Volume III. Here the row vector of prices, $P = [P_j]$, is determined by adding to cost outlays a constant percentage rate of profit or interest, r , reckoned on *all* cost outlays (wage payments plus raw material outlays), and is defined by

$$\begin{aligned}P &= [Wa_0 + Pa](1 + r) = Wa_0(1 + r)[I - a(1 + r)]^{-1} \\ &= WA_0(r) \\ Pm &= W\end{aligned}\tag{2}$$

Generally the solution of (2) involves solving an n^{th} degree polynomial for the appropriate positive root r^* , whereas (1) involves solving only a linear equation for s^* .

The Transformation Process. Generally, also, when confronted with a tableau of values in which the aggregates $\pi a = [\sum_i \pi_i a_{ij}]$ are not broken down into their

components, it is impossible to "identify" the underlying technical a coefficients and to infer unambiguously the equilibrium prices of (2). However, since 1907, Bortkiewicz and a long line of writers have proposed a "transformation" algorithm applicable to the special 3-industry model, in which industry 1 provides only intermediate goods (e.g. coal), industry 2 provides only subsistence wage goods for labor (e.g. corn), and industry 3 provides only luxury goods for non-labor (e.g. velvets). In this canonical model of "simple reproduction," we may as a convention define the observed industry outputs to be unity, $Q_j \equiv 1$, and total labor as unity. With this choice of units the system has the property

$$\begin{bmatrix} a_0 \\ a \end{bmatrix} = \begin{bmatrix} a_{01} & a_{02} & a_{03} = 1 - a_{11} - a_{12} \\ a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and the equations of (1) can be rewritten in terms of the Marxian categories of constant capital (c_j), variable capital (v_j), and surplus values ($s_j = sv_j$):

$$\begin{aligned} \pi_j &= \pi_1 a_{1j} + W a_{0j} + sW a_{0j} \equiv c_j + v_j + s_j \quad (j = 1, 2, 3) \\ c_1 + v_1 + s_1 &= c_1 + c_2 + c_3 \\ c_2 + v_2 + s_2 &= v_1 + v_2 + v_3 = W \\ c_3 + v_3 + s_3 &= s_1 + s_2 + s_3 = sW \end{aligned} \quad (3)$$

A celebrated example of such a values tableau is the following

$$\begin{aligned} 225 + 90 + 60 &= 375 \\ 100 + 120 + 80 &= 300 = W, \quad s^* = 2/3 \\ 50 + 90 + 60 &= 200 \end{aligned}$$

To "transform" this into a price tableau, Bortkiewicz defined transformation coefficients between prices and values in the form of the row vector $y = [y_j]$, defined by the relations

$$\begin{aligned} (y_1 c_1 + y_2 v_1)(1 + r) &= y_1(c_1 + c_2 + c_3) \\ (y_1 c_2 + y_2 v_2)(1 + r) &= y_2(v_1 + v_2 + v_3) \\ (y_1 c_3 + y_2 v_3)(1 + r) &= y_3(s_1 + s_2 + s_3) \end{aligned} \quad (4)$$

In order for these three homogeneous linear equations to have a nonzero solution, it is necessary that the first two of them have such a solution, which obviously requires the following determinantal quadratic to vanish:

$$0 = \begin{vmatrix} c_1/\Sigma c_j - (1 + r)^{-1} & v_1/\Sigma c_j \\ c_2/\Sigma v_j & v_2/\Sigma v_j - (1 + r)^{-1} \end{vmatrix} = (1 + r)^{-2} - b_1(1 + r)^{-1} + b_2 = 0$$

This can be solved for the relevant positive root, $1 + r^*$. For the above numerical tableau, it can be shown that $r^* = 1/4$ and $y_1/y_3 = 32/25$, $y_2/y_3 = 16/15$, so that the price tableau must be *proportional* to

$$\begin{aligned} 288 + 96 + 96 &= 480 \\ 128 + 128 + 64 &= 320 = W \\ 64 + 96 + 40 &= 200 \end{aligned}$$

How one scales or normalizes W and these numbers is an unessential issue even though it has given rise to some acrimonious and sterile debate among scholars.

Explication of the Transformation's Meaning. This completes the traditional transformation problem, which has frequently been regarded as a vindication of Marx's Volume I analysis. However, direct and simple substitution of (3) into (4) shows that the latter's Bortkiewicz algorithm is equivalent to solving

$$\begin{aligned} y_j \pi_j &= [W a_{0j} + \sum_{i=1}^n y_i \pi_i a_{ij}] (1 + r), & (j = 1, \dots, n) \\ \sum_{j=1}^n y_j \pi_j m_j &= W \end{aligned} \quad (5)$$

But (5) is seen to be precisely that of (2), with

$$P_i = y_i \pi_i, \quad y_i = P_i / \pi_i \quad (i = 1, 2, \dots, n)$$

Hence when value magnitudes such as $\pi_i a_{ij}$ are multiplied by y_i , *all that is being done is to cancel out the π_i values from the problem*—as in $y_i \pi_i a_{ij} = (P_i / \pi_i) \pi_i a_{ij} = P_i a_{ij}$!

In summary, "transforming" from values to prices can be described logically as the following procedure: "(1) Write down the value relations; (2) take an eraser and rub them out; (3) finally write down the price relations—thus completing the so-called transformation process." The present elucidation should not rob Marx of esteem in the eyes of those who believe a subsistence wage provides valuable insights into the dynamic laws of motion of capitalism.

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¹ Sweezy, P. M., *Theory of Capitalist Development* (New York: Oxford University Press, 1942), Ch. 7.

² Dorfman, R., and P. Samuelson, and R. Solow, *Linear Programming and Economic Analysis* (New York: McGraw-Hill, 1958), Ch. 10; Seton, F., *Review of Economic Studies*, 24, 149 (1957); Samuelson, P., forthcoming in the *Journal of Economic Literature*, 8 (1970).